Numerical and avalytical study for fractional differential equations.

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In the past three decades, the subject of fractional calculus (calculus of integrals and derivatives of arbitrary order) has gained considerablepopularity and importance, mainly due to its demonstrated applicationsin numerous diverse and wide spread fields in science and engineering.For example, fractional calculus has been successfully applied to prob-lems in system biology, physics, chemistry and biochemistry, hydrology,medicine and finance. In many cases these new fractional order mod-els are more adequate than the previously used integer-order models,because fractional derivatives and integrable enable the description ofthe memory and hereditary properties inherent in various materials andprocesses that are governed by anomalous di usion. Hence, there is agrowing need to find the solution behavior of these fractional di eren-tial equations. However, the analytic solutions of of the most fractionaldi erential equations (FDEs) generally cannot be obtained. As a conse-quence, approximate and numerical techniques are playing an importantrule in identifying the solution behavior of such fractional equations andexploring their applications. There are many versions of definitions forfractional -derivatives and integrals. We mention here, the formal defini-tion (Riemann Liouville) and its modified form (Caputo).The main objective of this thesis is to develop new e ective approximateixand numerical methods and supporting analysis for solving initial valueproblems of fractional order, boundary value problems of fractional order,FDEs with delay, systems of FDEs, a class of fractional variational prob-lems (FVPs) and a class of fractional optimal control problems (FOCPs).This thesis consists of six chapters:Chapter one. In this chapter, we introduce some definitions, lem-mas and important theorems, without proof, which are needed and usedthroughout this thesis.Chapter two. This chapter is devoted to study the numerical andanalytical treatment for the e ectiveness of operator method which isproposed to find analytical solutions for a certain class of FDEs. Ananalytical criterion is constructed for determining if there exists a solutionfor this class of FDEs in terms of exponential functions. Several examplesare used to illustrate the proposed concept.Chapter three. We investigate in this chapter a new -approximateformula to express the derivatives of any fractional order based on La guerre orthogonal polynomials. An efficient spectral collocation methodis introduced for solving multi-term FDEs with initial values. Also, aprocedure for solving fractional di usion equations is introduced. somenumerical examples are proposed to show the accuracy and efficiency ofthese approaches.Chapter four. In this chapter, a computational matrix method ispresented to nd approximate solutions of high order fractional di er-ential equations in terms of shifted Legendre polynomials via -Legendrexcollocation points and systems of high order fractional di erential equa tions in terms of shifted Chebyshev polynomials via Chebyshev colloca-tion points in the interval [0; L]. Illustrative real problems are given toshow that these approaches give satisfactory and accurate results.Chapter ve. In this chapter, an ecient numerical method to ob-tain approximate and exact solutions of the fractional delay -di erentialequations using Legendre collocation method is proposed. Exact solu tions for the proposed numerical examples show the e ectiveness of -thisapproach.Chapter six. This chapter addresses and investigates a new op erational matrix method which is based on a combination of shiftedChebyshev polynomials and nite di erence methods. It is applicablefor solving fractional boundary value problems (FBVPs), fractional de-lay boundary value problems, a class of fractional variational problemsFVPs and a class of fractional optimal control problems FOCPs. Thisproposed technique is based on using matrix operator expressions whichapplies to the di erential terms. An upper bound for error of the approxi-mate formula of the fractional derivatives using the proposed operationalmatrix method is obtained. To illustrate the accuracy of the proposedtechniques, several numerical examples are introduced.