maximum principle indifferential equations and its applications

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-The present thesis is concerned with the maximum principlesin differential equations. These principles provide us with one ofthe most useful and powerful tools in the qualitative andqualitative study of both ordinary and partial differentialequations. Since the early nineteenth century, the maximumprinciples for solutions of elliptic equations and inequalitieshave been used intensively in the mathematics literature.Throughout the last decades and till now, these principles havebeen refined and extended to more and more types of equationsassociated with different conditions on the boundaries of regions,where the equations are cons ide red [see references c it.ed at theend of this thesis]. One of the most important refinements, knownas the Hoof maximum principle, asserts that at a maximum on theboundary, the outward normal derivative of the solution of anelliptic or a parabolic equation' is positive.The aain objective of this dissertation is to give acompletesurvey on the maximum principles and their various versions forordinary and partial differential equations and some of itsapplications, We give also some examples illustrating t.he ideasdiscussed in this work.The thesis consists of four chapters. In chapter I we discussand prove different versions of the maximum principle for ordinarylinear differential inequalities and equations, and some of itsapplications to initial and boundary value problems. At the end of--_. -- - --- - -- _.. ----- -- -- --- - -- -.iFthischapter the case of nonlinear equations is investigated.In chapter II we introduce and discuss the maximum principlesfor elliptic partial differential equations and inequalities andsome of its generalizations. Applications of these principles to,different uniqueness problems are also given.In chapter III we consider solutions of the nonlinear'elliptic equation. 11 u + feu) = 0 and obtain bounds for variousquantities associated with this problem. We show that it ispossible to find functions g and h so thatP ~ 2 g(u) Igrand u I +h(u) satisfies an elliptic inequality and by applying a maximumprinciple we prove that P either attains its maximum on theboundary or at a critical point of the solution u.In chapter IV we give some applications of the material ofthe previous chapters. We study the torsion problem and the••efficiency ratio of a nuclear reactor.