
the discovery of neptune acritical max mination of the theory of levelrrier

mohamed kamel gaber baghdad

The main aim of LeVerrier's planetary theory was to determine the position of the disturbing planet, Neptune, from an analysis of the orbital motion of Uranus. In this respect he produced three solutions of increasing accuracy, classified as the First, the Second and the Third solution respectively, the goals and results of which are summarised below the First Solution aimed at determining the mass of Neptune on the assumption that the ratio, a , between the semi-major axes of Uranus and Neptune equals in keep-ing with Bode's Law. For this solution, eight equations of condition were solved corresponding to the years $1747.7 + 14(j-1)$, $j = 1, 2, \dots, 8$, covering both ancient and modern observations, but utilis- (i) CO (i)ing only those perturbatory forces P , N , M ($i=1, 2, 3$), of order zero and unity, in e and e' . According to this solution, the mass of the disturbing planet was of order 4700 times that of the Sun. The Second Solution, aimed at determining the orbital elements of the disturbing planet and refining the value for the mass, previously determined. For this solution three further equations of condition were added corresponding to the epochs 1690, 1712 and 1715 respectively. The remaining equations of condition were rearranged into fifteen alternative equations, at intervals of seven years, corresponding to the epochs $1747.7 + 7(j-1)$, $j=1, 2, \dots, 15$. On solution, and following close examination of the errors between theory and observation for the epochs 1690, 1747, 1758, 1793 and 1828, LeVerrier came to the conclusion that the mass of the disturbing planet should lie between 10,000 th and th that of the Sun, and deduced that the true heliocentric longitude on 1st January 1847 should be approximately 325, the Third and final solution, called upon the experience gained in the First and Second Solutions, and attempted to improve the accuracy of the values for the mass and the orbital elements of the disturbing planet. Using a range of values of a from 0.49 to 0.53, LeVerrier formed thirty-three equations of condition covering a period of 155 years, and for the first time included certain second-order terms. The solution, as we have seen, yielded a value for the mass equal to approximately 1 times that of the Sun and a predicted true helio-9300 centric longitude for 1847.0, less than one degree from its actual position in his investigation of the theory of Adams (1847) solutions have been obtained for different years before 1845, backwards in time to the year 1810 at intervals of 5 years, utilising only those observations that would have been available at that time. No firm conclusions can be drawn from the result obtained due to the frequent occurrence of unacceptably high values of eccentricity.

Nevertheless, there is perhaps some justification for stating that LeVerrier's technique would have led to a successful prediction in the years 1835 to 1845 for some values of a , notably $a = 0.49$ to 0.52 . Beyond these values, only one gave a sensible solution, viz. $a = 0.60$. Comparison of the predicted positions corresponding to the theory of both Adams (Brookes) and LeVerrier (Baghdady) is possible now, for the first time. the two analyses have several epochs and values of a in common, and permit a direct comparison for values of $a = 0.49, 0.50, 0.52, 0.53, 0.56, 0.57, 0.60$ (approximately), for the years 1846, 1840, 1835, 1830 and 1825. The results are presented in Tables 7.1 and 7.2, the letters A and L signifying successful predictions according to the theory of Adams and LeVerrier respectively. As in Section 6, predictions yielding values of $e' > 0.2$ are regarded as unsuccessful, and consequently are not included in these tables.

a	0.49	0.50	0.52	0.53	0.56	0.57	0.60	1846	1840	1835	1830	1825
Adams	-L	-L	AL	AL	-L	-L	AL	AL	AL	AL	-L	A
LeVerrier	-L	-L	AL	AL	-L	-L	AL	AL	AL	AL	-L	A

Table 7.1 (within ± 5)

a	0.49	0.50	0.52	0.53	0.56	0.57	0.60	1846	1840	1835	1830	1825
Adams	AL	AL	AL	AL	AL	AL	AL	AL	AL	AL	AL	AL
LeVerrier	AL	AL	AL	AL	AL	AL	AL	AL	AL	AL	AL	AL

Table 7.2 (within ± 10)

105 -It would seem, therefore, that LeVerrier's technique proves the more successful within the range ± 5 , whilst that of Adams proves more successful within the range ± 10 . Both techniques are particularly unsuccessful when applied in 1830 and only partially successful in 1825. Of the two theories, that of Adams came closest to the correct value of eccentricity, viz. $e' = 0.0088$ (for $a = 0.60$ in 1846) compared to the actual value of 0.0086 . On the other hand, e' was always > 0.2 for the year 1830. Values of e' according to the theory of LeVerrier were consistently out by a factor of at least ten, the fortuitous timing of the predictions in 1846 is thus self-evident, especially if one recalls the usage of erroneous values of a . However, the validity of both theories seems established, since the best solutions (in terms of a' and e'), are those corresponding to the highest value of a , viz. $a = 0.60$ ($a' = 32$ A.U.), which is not far removed from reality, viz. $a' = 30$ A.U.